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ABSTRACT

Presented are examples of student differences in thought forms derived from interviews of students entering the natural science line and technology line of the integrated upper secondary schools in Sweden. Methods are outlined for the symbolic description, analysis and interpretation of "thought forms," and ways are described in which dialogue between teacher and student can be used for diagnosing the thought forms that come into play during the teaching of a certain subject content. (CS)

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CONCEPT FORMATION IN MATHEMATICS AND THE NATURAL SCIENCES; THE INTEGRATED UPPER SECONDARY SCHOOL (10)

SOME REAL PICTURES FROM THE CLASSROOM-BASED RESEARCH IN PHYSICS TEACHING AT GÖTEBORG

Paper prepared for the 'Cognitive Development Research Seminar, Science and Mathematics Education; Session 15: Classroom-based research',
The University of Leeds, 17-22 September 1979.

Leif Lybeck

No 88 November 1979

INSTITUTE OF EDUCATION
UNIVERSITY OF GÖTEBORG

Abstract

The research approach in Science and Mathematics Education at Göteborg has been presented in another paper at this conference.
In this paper, examples will be given of results related to different problem areas in the theoretical model. The step-by-step procedure in the studies based on classrooms and laboratories will be demonstrated with empirical evidence.

The research reported here has been financed by the National Swedish Board of Education

Introduction

The theoretical reflections on our research approach in the field of Science and Mathematics Education at Göteborg have been presented by Lybeck (1978a, 1979a, c).

Many of the students in the natural science (N) line and technology (T) line of the integrated upper secondary school (grades 10-12, ages 16-18) experience a dramatic change in the way the world around them is perceived as a result of the transition from the nine year compulsory comprehensive school. Physics and chemistry present abstract concepts that take a long time to understand. In spite of the fact that the students are interested in the natural sciences and mathematics, they are confronted by a way of reasoning that many of them have little experiences of; at the same time they are required to reason along very definite lines. The question is:

> How do the students reason when they begin their studies in the N and T line and
> what changes take place in their reasoning during their studies?

We believe that partial answers can be given if one studies the way in which students reason when they are working with a certain subject content, i. e. a particular scientific phenomenon related to a principle or certain concepts, e. g. Archimedes' principle and the concepts of density and pressure. We try to describe the students' qualitatively different conceptions of a phenomenon at a certain point in time and during the teaching itself. The object of the studies consists of the spontaneously expressed qualitative conceptions or thought forms. The object of our studies cannot be found in textbooks or in the teacher since it is a phenomenon that acts as a relation between the student and the subject content studied by the student.

In individual interviews, I presented information that is intended to illustrate the N and T students' understanding of concepts at the beginning and the end of the first school

year (grade 10, ages 16-17). Let us exemplify the results with the Measuring Cylinder Task introduced by Suarez (1977 (1975)) without touching on the actual interview procedure.

The Measuring Cylinder Task

Figure 1. Starting-point in the Measuring Cylinder Task.

Figure 2. Question 4-y in the Measuring Cylinder Task.

Identically graduated measuring glasses with different diameters are used. The scales are not numbered nor is any unit of measurement given. First, the interviewer fills the thick cylinder with water up to level '6'. When the water is poured into the thin cylinder it rises to level '9' as shown in Figure 1. This is the starting-point. The subquestions are now put.

Question 4-y E: If I pour up to 4 in the thick cylinder, how high would it come in the thin cylinder (which is empty)?

Question 9-y E: If I pour up to 9 in the thick cylinder, and then pour the water into the empty cylinder, how high would it come?

The answer is followed up by:

E: How did you work it out?

Well, how would you, dear reader (teacher), reason when you

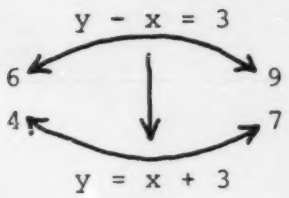
solved the problem?

This is how the students 5N and 4N (N line, girl and boy) answered:

S: Yes, I thought like this, the difference between 6 and 9, I thought then, (y - x = 3)
and so 'plussing', adding ... you add it, the difference to 4. ... 7. (5N) (y = x + 3)

S: 7. (4N)
E: How did you arrive at that?
S: 6 minus 4 is 2. (x - y = 2)
9 minus 7 is 2. (x - y = 2)

Our algebraic interpretation is given after the students' answers. We use x and y in the conventional sense as first and second term in the pair of values (x, y). The students arrived at the same answer but used completely different solving strategies which we have described symbolically as follows:



(Student 5N)

(Student 4N)

The two-dimensional table of categories

We have found that the best interpretation of a nuanced, empirical body of material is provided by a general line of thought which reflects the concept of function and an interacting conception of how quantification should take place. We call the line of thought the function aspect of the students' conception of the proportion and proportionality tasks which have the answer categories A and B. The quantification

aspect stands for the way in which mathematical instruments are applied and has four categories, 1-4. (Lybeck 1978b, 1979b). Thus, student 5N quantifies between the relevant variables (A) and student 4N within one variable at a time (B).

Note that 5N utilizes the form in the relation that explicitly expresses the number she wants to obtain. 4N, on the other hand, does not change forms when he applies the relation for the second measuring glass and consequently does not explicitly express the unknown size. These ways of quantifying belong to answer category 4 (see below, p 9). The thought forms used by the students are called A4 and B4 in the two-dimensional table of categories. At the beginning of the school year, 4 students answered question 4-y with the A4 form and 1 out of 26 with the B4 form; at the end of the school year, 9 students out of 76 answered the question with the A4 form. In spite of one school year of studies spontaneously expressing the 'corrective' reflection: 'If I pour from the empty thick cylinder into the thin one, then it will be filled up to the third scale line' does not come naturally to these students.

Within each of the function aspect's category of answers A and B the students express the changes that have taken place in different ways. We also find, however, the same mathematical instrument among the answers belonging to A or B. Below are the four categories of answers referring to the quantification aspect.

The quantification aspect with examples of students' answers

1. Direct proportion. The relationship is expressed as a multiplication or a division or as a relation (ratio), e.g. as in

   $y = 1.5 \cdot x$ or $\frac{y}{x} = \frac{3}{2}$

   or inverted as in

   $x = \frac{y}{1.5}$ or $\frac{x}{y} = \frac{2}{3}$

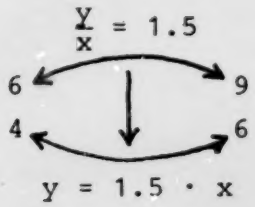
Student 10N produces a thought form from category A.

S: I divided 9 by 6. ($\frac{y}{x} = 1.5$)

E: Yes.

S: And I multiplied what I then got, 1.5 by 4. (10N) ($y = 1.5 \cdot x$)

He only uses the form in the relation that explicitly expresses the number he is looking for. The solving strategy can be described symbolically as follows.



(A1 form)

Student 9T (T line) quantifies within the category B:

6-9
3-4,5 (9T's note)

E: ... Why do you do that?

S: Half. .... Yes. That's what it says ($y = \frac{x}{2}$)
... 3, so it ought to be 4.5.
... It says 4 (the thick cylinder).

E: 4, yes. And then, it was as I asked (question 4-y).

1,5
4 (9T's note)
($y = x \cdot 1.5$)

S: 6.

E: Did you obtain 1.5.

S: Yes.

E: How did you obtain it?

S: Divided 4.5 by 3. ($y = \frac{x}{3}$)

E: Oh! Why did you divide by 3?

S: In order to get how much it is per step.

His solving strategy can be described schematically as

follows:

$$y = \frac{x}{2} \quad \overset{6}{\underset{3}{\rightleftarrows}} \quad \overset{9}{\underset{4.5}{\rightleftarrows}} \quad y = \frac{x}{2}$$

$$y = \frac{x}{3} \quad \overset{}{\underset{1}{\rightleftarrows}} \quad \overset{}{\underset{1.5}{\rightleftarrows}} \quad y = \frac{x}{3} \qquad \text{(B1 forms)}$$

$$y = 4 \cdot x \quad \overset{}{\underset{4}{\rightleftarrows}} \quad \overset{}{\underset{6}{\rightleftarrows}} \quad y = 4 \cdot x$$

The student does not tackle the problem directly in this way.

$$\frac{y}{x} = \frac{4}{6} \quad \overset{6}{\underset{4}{\rightleftarrows}} \quad \overset{9}{\underset{6}{\rightleftarrows}} \quad y = \frac{4}{6} \cdot x$$

He introduces two hypothetical volumes to help him. This is a consequence of the difficulties involved in a multiplication by $\frac{4}{6}$.

From a methodological viewpoint it is important to note the essentially different thought forms illustrated by the answers given by 10N and 9T. Category A of the function aspect leads to the algebraic form $y = 1.5 \cdot x$, i. e. the general functional relationship for direct proportionality, while category B leads to quantitative algebraic forms that do not express any general direct proportionality. This constitutes didactic knowledge that is of great importance to concept formation in the natural science subjects, where relevant variables of different qualities lead to the new concepts by means of the A1 form. It is the A1 form that is developable.

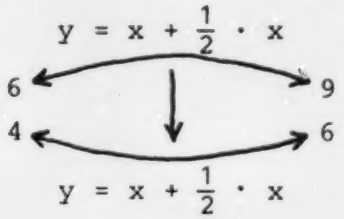
2. Proportional increase or decrease. The relationship is expressed as an addition, subtraction or difference with the aid of a (increment) factor or ratio, e. g. as in

$$y = x + \frac{1}{2} \cdot x, \quad y - x = \frac{1}{2} \cdot x \text{ or } \frac{y - x}{x} = \frac{1}{2}$$

or inverted as in

$x = y - \frac{1}{3} \cdot y$, $y - x = \frac{1}{3} \cdot y$ or $\frac{y - x}{y} = \frac{1}{3}$

Student 9N quantifies between the measuring glasses (A).



(A2 form)

S: Or .... yes, at 6 in the thick cylinder it made 9 when poured over. So it is, of course, half of 6 one adds. ($y = x + \frac{1}{2} \cdot x$)

E: I see !

S: And since it was at 4 there, I add 2 so it'll be 6. (9N) ($y = x + \frac{1}{2} \cdot x$)

We find similar ways of quantifying among students whose answers have been placed in category B.

S: Now it has diminished by 2. It is of course, $\frac{1}{3}$ of 6. And $\frac{1}{3}$ of 9 as it was then, it is, of course, or it will be $\frac{2}{3}$, it is 6. ($y = x - \frac{1}{3} \cdot x$) ($y = \frac{2}{3} \cdot x$)

E: Well, yes. Maybe you would like to have a pen?

S: Eh. 6 minus 4, that makes 2. ($x - y = \frac{1}{3} \cdot x$)

E: Yes.

S: 2 over 6 is equal to $\frac{1}{3}$ and 9 , yes, $\frac{1}{3}$ times 9, that makes 3 then. Then you take minus 9, no, 9 minus 3. 6. (1N) ($\frac{x - y}{x} = \frac{1}{3}$) ($y = x - \frac{1}{3} \cdot x$)

Student 1N first expresses the relation with the aid of a decrease ($y = x - \frac{1}{3} \cdot x$). Then, when he writes down his thoughts he gives $\frac{1}{3}$ explicitly via the form $\frac{x - y}{x} = \frac{1}{3}$. When he is to use the relation in the other measuring glass, he alternates between the direct proportion ($y =$

$= \frac{2}{3} \cdot x$) and the proportional decrease ($y = x - \frac{1}{3} \cdot x$), but finally chooses the latter form. In both cases the unknown number is explicitly expressed. His method of solving the problem can be described symbolically as follows:

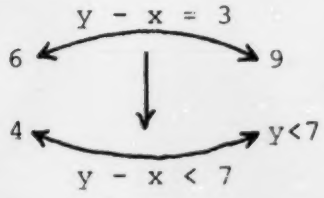
$$\frac{x - y}{x} = \frac{1}{3} \quad \begin{matrix} 6 & \longrightarrow & 9 \\ 4 & & 6 \end{matrix} \quad y = x - \frac{1}{3} \cdot x \qquad \text{(B2 form)}$$

3. <u>Relating either absolute increase, absolute decrease, or absolute difference</u>.
   The student realizes that the increase, decrease, or difference cannot be absolute (see category 4 below), but the student does not succeed in quantifying the relationship in categories 1 and 2 above.

Student 6T's answer will serve as an example of category A.

S: ... no, not quite 7, close to 7, I think.
6,5, I think.
E: Nearly 7? What makes you think that?
S: Well, I don't know ...
E: Yes. Can you tell me how you reasoned?
S: Eh ... if 6 and 9, so it was three apart. ($y - x = 3$)
4 is less and then, of course, it can't be 3 apart. ...
Yes. It can't be quite 3, now when it's less (the water in the thick cylinder). (6T) ($y - x < 3$)

His solving strategy can be described as follows:



(A3 form)

Like student <u>5N</u> (above, p 3) <u>2N</u> answers <u>question 4-y</u> in the A4 form, i. e. a quantification by means of an absolute increase between the measuring glasses. <u>2N</u> considers

question 9-y.

S: Hm. Yes. There must be some connection.
E: There must be some connection, yes.
S: If you have more in the thick one, then it would increase more in the thin one than if you had less in the thick one and poured it into the other one. (2N) (B3 form)

She relates an increase in one of the measuring glasses to a larger increase in the other, i. e. a B3 form. She considers the answer to question 9-y to be 14 or 15 and now has the opportunity of checking her hypothesis experimentally. She tabulates the pairs of figures (4, 6), (6, 9) and (9, 13.5) and describes the relation in accordance with thought form B2 as follows:

S: It increases 1½ per step. (B2 form)

4. Quantification through absolute increase, decrease or difference.
The relationship is seen as an addition, subtraction or a difference as in $y = x + 3$ or $y - x = 3$ or inverted as in $x = y - 3$.

Students 5N and 4N (above, p. 3) used thought forms in accordance with A4 and B4 respectively in their answers.

We cannot do the students justice in this paper by presenting the wealth of thought forms used by them but must instead refer to the BMN project's reports. In conclusion, however, let us illustrate how student 3T describes the inverse thought form ($x = \frac{2}{3} \cdot y$).

S: There is $\frac{2}{3}$ in the large ... of what will be in that one (the thin one).

However, he solves question 9-y with the A2 form $y = x + \frac{1}{2} \cdot x$, probably because of the arithmetic complexity that arises when the thought form in A1 is retained.

The concepts of proportion and proportionality can be seen as theme in the physics' and chemistry's syllabi for the N and T lines, grade 10. One of the BMN project's purposes was to study how the students quantified in above all the physical concept formation process. We can now in the following study recorded events in classrooms and laboratories with the help of the two-dimensional category table.

The thought forms in the classroom

The fact that some of the thought forms described in the table come into play during teaching can be demonstrated by what happened during a follow-up of laboratory work on density in the N line, grade 10. The recording was made during the pilot studies and the subjects were not the same students as those interviewed in accordance with the category table. The teacher wrote the following value table on the blackboard. It has nothing to do with the values observed since the teacher's intention was to make the students conscious of the concept of proportionality. He wanted to introduce the concept of proportionality as an instrument in the formation of physical concepts.

x	y
2	3
5	7.5
6	9
8	12
12	?

T: If I point to 12 in the table, can you tell me what I should write in the y column?

S: 18.

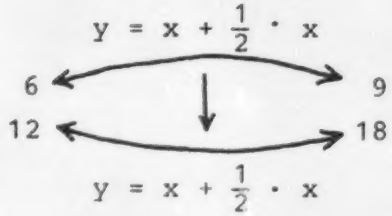
T: How do you know?

S: You look at all the numbers. You divide the x number and then you take the x number plus what was divided (i. e. $\frac{x}{2}$). That's how you get it.

T: Once again.

S: If you take the x number and divide in the middle, then you take the x number plus the number divided. That's how it goes.

This student's thought form can be symbolized in the A2 form.



(A 2 form)

The table as an analysis instrument

The two-dimensional category table is a result of the analysis of the transcribed interviews. This constitutes didactic knowledge formation. Let us now demonstrate how the table can be used as an analysis instrument in studies of transcriptions of recordings of lessons. A very clever student in the N line, grade 10, was confronted with Archimedes' problem concerning a floating object in water (Lybeck (1979b)).

Tor: Yes, if you take the bit that's under water you can measure it and see what its volume is. And then you get, for example 3 cm^3. Then it has displaced 3 g of water and then you measure the rest of the piece above the water to see how much of it there is. So if one says that 1/3 is under water, then one has to take that weight times 3 in order to get the weight of all of it.

And: No.

...

T: They want you to repeat what you said.

Tor: Yes, but the piece of the box that is below the water surface, it has a certain volume then, say 3 cm^3. And then 3 cm^3 of water is displaced. And that'll be 3 g of water too. And then that piece that's not below the water (surface). One has to calculate a percentage, how much of

the piece that is below the surface in order to get the whole, and then it weighs 3 g, the piece under water. And then, the rest ... then one knows what part of the box weighs and then all one has to know is how large the part is one has weighed.

E: May I ask whether the part which is below the water surface is 1/3 of the whole (box)?

Tor: Yes.

E: It weighs 3 g, you said?

Tor: Yes.

E: That was your example?

Tor: Yes.

E: How much does the whole thing weigh?

Tor: It weighs 9 g, then, yes.

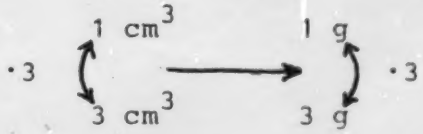
E: I see.

....

Van: That can't be right.

T: Do you agree with And?

We interpret Tor's conceptions as follows. He perceives the concept of density in the B1 form.



This qualitative conception of the concept of density brings to mind the 13-14th centuries concept 'specific weight', which is the weight of an appropriate chosen volume in an experimental connection. There is thus a conceptual distinction between weight and specific weight; they are, however, measured in the same physical dimension, namely weight, in this 'Aristotelean world of conceptions'. In this hypothetical, experimental context, Tor makes this assessment in B1 forms a perceptually estimated volume and finally to the object's total weight (in air). (See figure 3).

Tor was completely convinced that the 'strength in the interaction' between the subsystems 'water' and 'floating object' is a measurement of the part of the object

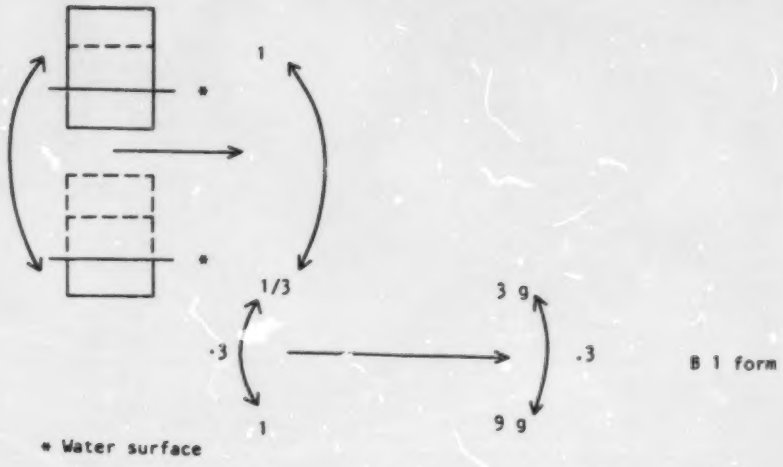


Figure 3. Interpretation of Tor's conceptions.

below the surface of the water. The researcher's (E) questions are an example of how empirical evidence is generated in order to confirm the didactic hypothesis regarding Tor's qualitative conception of the phenomenon. Some of his class-mates disagreed with him. They tried to make him abandon his conception by means of different arguments but he remained faithful to his 'theory'. The cognitive conflict created in Tor and several of his comrades was solved the next day when they discovered Archimedes' principle, which was then formulated in a more suitable way from the point of view of physics, by means of experiments planned and executed by them.

Tor: The weight of the object is equal to the weight of the displaced water.

Tor's problem at the senior level of the comprehensive school

The didactic problem which Tor and his teacher helped us to formulate has been given to several classes at the senior level of the comprehensive school, grades 7 and 8. Here is a similar extract from a recording made in grade 7 at the beginning of the spring term after the concept of density had been introduced.

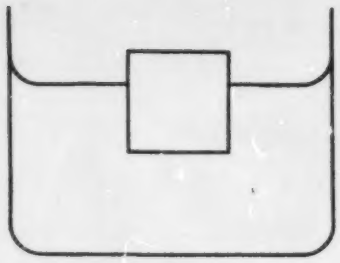


Figure 4. Beaker filled with water and a floating wooden block.

T: ..... You are to estimate how much of it sinks.

Rolf: About 3/4.

T: 3/4, yes. 3/4 below the surface of the water, 1/4 above. Is there anyone who can say what the density of the wood is, then? What is the density of this piece of wood? This contains ordinary water now.

Rolf: 1.25.

T: How did you reason?

Rolf: I reasoned, but I reasoned incorrectly. But I reasoned that the water had 1 gram per cm^3 in density, so the wood had 1/4, yes, I should say less,

T: Hmm. The wood had 1/4 less, you said. What would it have been then?

Rolf: 0.75 g. (N. B. He does not say grammes per cm^3.)

T: 0.75. What would have happened if the piece of wood had had a density of 1.25?

Bo: It would have sunk.

We have interpreted Rolf's thought forms symbolically as follows with the analysis instrument based on the students' proportion and proportionality way of reasoning. (See figure 5).

It is probable that Rolf means the weight of a certain volume chosen for the purpose, namely 1 cm^3, when he says '1 gramme per cm^3' density. He has quantified with the B2 form. We note that he still understands the concept of density in a B form and evidently not in an A form as

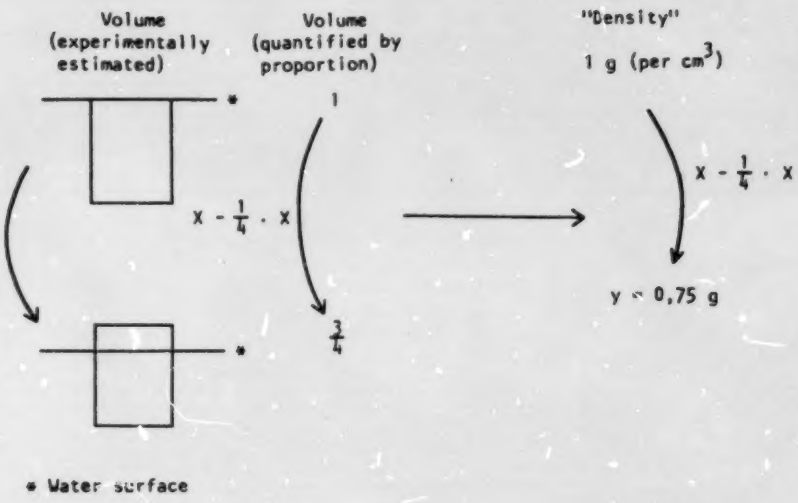


Figure 5. Interpretation of Rolf's conceptions.

his first statement could have been interpreted from the perspective of physics.

Which concept of density do the students take with them to the upper secondary school? Surely, density is expressed in the A1 form $\varrho = \frac{m}{V}$ at the senior level? This is certainly the case, but, as our recordings from the senior level show, the concept is given a meaning mainly in the B form.

Student-teacher dialogues

After a teacher has introduced the concept of density in the A1 form in a class, grade 7, at the senior level of the comprehensive school, the teacher and the pupils together solve a couple of density problems. In a concrete example, the mass was 200 g and the volume 20 cm^3 and in another 70 g and 10 cm^3 respectively. One pupil thought that 200 should be divided by 20. At this point, we join the conversation.

S:	It's <u>per</u> cm^3.	(Invitation to A1 form?)
T:	Yes, I'll work out <u>how large the mass is for each cm^3</u>. What is the mass of 1 cm^3 in the first case, then, with this object?	(B form for the density.)

(200 g and 20 cm^3 respectively)

S: 10.

T: Yes, it will be 10 g per cm^3, as I usually express it. Each cm^3 has a mass of 10 g.

(The B form does not require the implied unit to be stated.)
(The pupil's B form is met by an A form from the teacher?)
(The teacher explains in the B form.)

T: What comparison figure do we get over there? (70 g and 10 cm^3 respectively.)

S: 7 g per .....

(The unit cm^3 disappears from the 'denominator' since 1 cm^3 is implicit in the B form.)

T: I get 7 g.
The other object weighed 7 g <u>for every</u> cm^3.

(The B form is established.)

In order to avoid any misunderstanding, I would like to mention that the B form is naturally not an incorrect thought form. In fact, it has even been used as a form of knowledge in physics. The problem is rather that this qualitative conception is not as fruitful as the A form has shown itself to be in modern physics. The teacher is quite correct when he uses B forms for the concept of density, and 'for every' is a correct model to use. But one should not leave it at that in the belief that it provides an understanding of the concept in the A1 form. In such a case, the pupil will be faced with a blind alley with regard to concept formation. (We wish to note the discussion held by Arons, (1976).) We invite the reader to make his own interpretation of this dialogue which has been taken from a context that cannot be given full justice here. The teacher concludes the section with the following words:

T: Now we have arrived at the point where we shall <u>compare equal amounts</u>.
And I have also 'developed' it so

(The appropriate volume: 'for every'.)

that we have obtained a
measure here of what the
mass of 1 cm^3 is.
And it is the comparison figure.
We usually call it density,
which is what so many of us here
have been talking about.

When the teacher says 'equal amounts' he means quite simply equally large volumes, i. e. the appropriate chosen volume in comparisons between different matters. The expression can give rise to discussions on the interaction between chemistry and physics teachers. The measurement obtained in the form of a comparison figure (note figure) is the density in the B form, namely, 'what the mass of 1 cm^3 is'. The teacher has built up a methodical strategy without knowing about the A and B forms. His great experience has shown him that pupils acquire a certain lasting understanding of the concept of density if B forms are used. From the teacher's perspective, his teaching functions satisfactorily. During laboratory lessons when the teacher walks about talking to groups of pupils, he explains his didactic conception to some of the pupils without having the researcher's knowledge at the time of the experiment itself. We reproduce the following:

T: Yes, I would probably want to have the last one. Volume divided by mass, if you learn it by heart, you may quite simply not be able to remember after a couple of months - was it volume divided by mass, or - mass divided by volume? What is important is that you learn to understand it all. What do we mean by saying that we want to find out about density? Well, density is, we extract a small volume unit of 1 cm^3 and then one can obtain its mass, that is, 1 gramme.

If you understand this you know that you must always divide by the volume, that's the point, the point is to come down to one cm^3 and see what it weighs.

S: What you get, then, is the density, isn't it?

T: Yes, it's the comparison figure between the different ty-

pes of matter.

S: Then for us the density was 3.

We have attempted to illustrate by means of the examples given above the learning and teaching phenomena from actual teaching where the pupil, the teacher and the pupil-teacher interaction has shown the phenomenon of the pupil's concept formation. The examples refer to the levels 1 and 3 in the model of the areas presented by Lybeck (1979c). Let us now give a couple of examples from textbooks in order to supplement the pictures painted above.

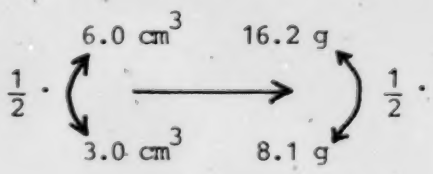
Educational material illustrated by the analysis instrument

In a recently published textbook for N- and T-students, grade 10 (ages 16-17), fitted to the new course of study in physics, the authors (Staffansson, Andersson, Johansson and Snaar, 1978) introduced the concept of density as follows:

'2.9 Density

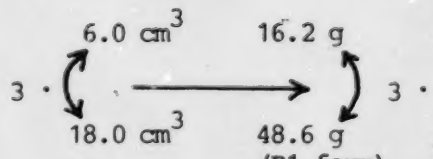
We weigh a number of bodies made of solid aluminium and determine their volumes by lowering them into a measuring cylinder filled with water. Our experiment tells us perhaps that 6.0 cm^3 of aluminium weighs 16.2 g. Half the volume, 3.0 cm^3, has half the mass, 8.1 g;

Our interpretations:



(B 1 form)

for a volume three times as large, 18.0 cm^3, the mass is three times as large, 48.6 g, and so on.



(B1 form)

If the mass increases in this way we say that the mass is proportional to the volume.
This is written
$m \propto V$
The relationship between the mass and the volume can also be expressed by the equation
$m = \rho \cdot V \qquad (2.1)'$ (A1 form)

(The students now have to leave the 'experimental context' and enter the 'theoretical context'.)

In the experimental study, i. e. the concrete situation, the B1 form is used. In the abstract (theoretical) context, one shifts to a qualitatively different thought form, namely, the physical law (the functional relationship) in the A1 form. The same methodical presentation is used when the dynamometer is introduced. This situation is valid for other educational materials and other subject contents.

Educational material used in mathematics can be analysed in a similar way. We cannot go into this here but would like to mention the fact that some of the educational aids at the senior level follow a B1 strategy in the general courses while B1, A1 and A2 strategies are applied to the methodic presentation of special courses.

It is one thing for our insight to be taken into consideration in the design of educational aids. But whether the A and B forms come into play during teaching, where educational aids are only one part of the process, is another matter entirely. In the latter case, the constructors of syllabi and courses of study as well as teachers and teacher trainers have a responsibility.

The didactic development of knowledge

Perhaps the BMN project's most important result is to have demonstrated how researchers in close contact with students and teachers can acquire didactic knowing and

and knowledge. Our aim has been to paint real pictures of learning and teaching with the help of descriptions from the classroom. It is my opinion that real pictures should be able to stimulate a discussion where the teacher with his intimate knowledge has much to contribute. It is important that the teacher participates since he himself is also the object of the study. We want to use our approach to form real pictures focused on the subject content imparted.

Our results show that the subject content is of far greater significance for the students' thoughts than had previously been considered the case. If the A1 form is used, then the student is, according to Piaget, a formal thinker. But we have found that this is not necessarily the case with regard to other tasks (e. g. the Spring Balance Task (Lybeck, 1978b, 1979b)) given to the students. Can this be a meaningful way of classifying students? Not in our opinion. We do not wish to classify the students. Our aim is to describe possible thought forms that may or may not come into play during the teaching of a certain subject content. It is descriptions of this type that ought to be part of the teacher's 'theory of teaching'.

The interview technique applied shows how a student's spontaneous thought forms, that are related to a certain subject content, can be found by means of a dialogue, i. e. how a dialogue can be used as a diagnosis. A new task in the schools is, for example, the planning of individual courses of study.

If the different thought forms concerning the subject content are mapped, then it will be possible to adapt the subject content to the student's need. If the students' thought contents are identical to the teaching content, then we have a possible link with the curriculum's aim with regard to the individual's intellectual development. Thus, if the students' thought forms

concerning a certain subject content are known and come into play during the actual teaching, then there exists one of the conditions for the student becoming an integrating human being instead of a human being integrated in the school system.

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